Alternative Model for Wall Effect in Laminar Flow of a Fluid through a Packed Column

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This paper has been written to complement the paper by Cohen and Metzner (1981). These authors developed a parallel channel model of the packed column which they regarded as a porous medium divided into wall, transition and interior regions. Their model is comprehensive, but involves a substantial empirical content. In the present paper an alternative model is proposed. This model is simple and is somewhat idealized, but it has a rational basis, and it leads to results which highlight the twofold nature of the wall effect.

As Cohen and Metzner note in their literature review, many authors treating the wall effect have emphasized one of its two aspects at the expense of the other. One group of authors have noted that the porosity of a packed column is of necessity higher in the immediate vicinity of the confining wall, where the solid particles cannot pack so well, than in the interior of the column. This allows a freer movement of liquid near the wall, and thus leads to a larger mass flux, for a given imposed pressure gradient, than would otherwise occur. A larger group of authors have dealt with the increased friction at the wall, which leads to a smaller mass flux, for a given pressure gradient, than would occur in the absence of the wall. The no-slip requirement at the wall has a restrictive effect on the flow.

TWO-REGION MODEL

In our model we suppose that a porous medium, wherein Darcy's law is applicable, occupies all the column except for a thin layer of uniform thickness adjacent to the wall. In this layer the porosity is taken as unity; i.e., the layer is occupied by fluid only. The thickness of the layer is taken to be of the order of a typical particle diameter, which is of the order $k^{1/2}$.

To be explicit, we consider a column filling a circular cylinder, with wall at $R = R_c$. The porous medium occupies the region $R \leq R_m$, where $R_m = R_c - \gamma k^{1/2}$, where γ is a numerical constant of order unity. We suppose that at the interface $R = R_m$ between the porous medium and the pure fluid, the pressure is continuous, and the velocity satisfies a boundary condition, introduced by Beavers and Joseph (1967), which is now widely accepted. [For this condition there is substantial experimental evidence, while some theoretical support has been provided, notably by Saffman (1971).] Because the pressure is continuous, it follows that the pressure gradient within the porous medium has to be equal to that within the pure fluid.

Although Cohen and Metzner treated the case of a power-law fluid as well as the case of a Newtonian fluid, we shall consider only the latter, for the sake of simplicity. Our governing equations are thus

$$\frac{\Delta p}{L} + \mu\left(\frac{d^2U}{dR^2} + \frac{1}{R}\frac{dU}{dR}\right) = 0 \quad \text{in} \quad R_m \leq R \leq R_c, \tag{1}$$

$$\frac{\Delta p}{L} = \frac{\mu V}{k} \quad \text{in} \quad 0 \leq R \leq R_m. \tag{2}$$

We have to solve the differential Eq. 1 subject to the boundary conditions

$$U = 0 \quad \text{at} \quad R = R_c, \tag{3}$$

$$\frac{\partial U}{\partial R} = \frac{\alpha}{k^{1/2}}(U - V) \quad \text{at} \quad R = R_m. \tag{4}$$

To make the subsequent algebra easier to handle, we introduce dimensionless variables, based on length scale R_c and velocity scale $\tilde{U} = R_c{}^2\Delta p/4\mu L$, and write $r = R/R_c$, $u = U/\tilde{U}$, $v = V/\tilde{U}$. Equations 1, 3 and 4 become

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + 4 = 0, \tag{5}$$

$$u = 0 \quad \text{at} \quad r = 1, \tag{6}$$

$$\frac{\partial u}{\partial r} = \lambda u - \beta \quad \text{at} \quad r = 1 - \epsilon, \tag{7}$$

where

$$\lambda = \alpha R_c k^{-1/2}, \quad \beta = 4\alpha k^{1/2}/R_c \quad \text{and} \quad \epsilon = \gamma k^{1/2}/R_c. \tag{8}$$

The solution to the system 5, 6 and 7 is

$$u = 1 - r^2 + A \log r, \tag{9}$$

where

$$A = \frac{\lambda(\eta - \eta^3) - \beta\eta + 2\eta^2}{1 - \lambda\eta \log \eta}, \quad \eta = 1 - \epsilon. \tag{10}$$

The volume flux through the cylinder is therefore

$$Q = \int_0^{R_m} 2\pi RV\, dR + \int_{R_m}^{R_c} 2\pi RU\, dR$$

$$= 2\pi R_c^2\tilde{U}\left[\int_0^{\eta} rv\, dr + \int_{\eta}^{1} ru\, dr\right]$$

$$= \pi R_c^2\tilde{U}[\eta^2 v + \tfrac{1}{2}(1 - \eta^2)^2 + A(-\tfrac{1}{2} + \tfrac{1}{2}\eta^2 - \eta^2 \log \eta)].$$

We now perform an expansion in powers of the small parameter $k^{1/2}/R_c$, taking into account that $v = 4k/R_c^2$, and neglecting fourth and higher powers. We find that

$$Q = \pi R_c^2\tilde{U}v\left[1 - \frac{\gamma k^{1/2}}{R_c}F(\alpha,\gamma)\right], \tag{11}$$

where

$$F(\alpha,\gamma) = \frac{12 + 18\alpha\gamma + 8\gamma^2 + 5\alpha\gamma^3}{6 + 6\alpha\gamma}. \tag{12}$$

We recognize that $R_c^2\tilde{U}v$ is Q_0, the volume flux which we would have in the absence of the wall, and that $\gamma k^{1/2}$ is the thickness d_f of the pure fluid layer. We can therefore write

$$\frac{Q}{Q_0} = 1 - \frac{d_f}{R_c}F(\alpha, d_f/k^{1/2}). \tag{13}$$

We observe that $F > 0$, and hence $Q < Q_0$, for all values of α and γ. Our model thus predicts the dramatic result that, even with a wall layer of porosity unity, the wall layer always has a restrictive effect, leading to a *reduction* in flux for a given pressure gradient. This reduction in flux is in accordance with the theoretical work of Cohen and Metzner and the bulk of the experimental work discussed by them. However, it seems that in some circumstances it is possible to get an increase in flux, as in the work of Furnas (1929) quoted by Cohen and Metzner. To account for this possibility we modify our two-region model, making it a three-region one.

THREE-REGION MODEL

We now introduce an intermediate layer, whose permeability exceeds the permeability of the bulk of the porous medium. We now have three regions: (I) a central core, $R < R_c - d_f - d_m$, occupied by a porous medium with permeability k; (II) an intermediate region, $R_c - d_f - d_m < R < R_c - d_f$, occupied by a medium with permeability k_m, where $k_m > k$; and (III) a wall region, $R_c - d_f < R < R_c$, occupied by pure fluid. As before, we suppose that d_f is of the same order as a particle diameter. The thickness d_m of the intermediate region is assumed to be equal to several particle diameters.

The (superficial) velocity V_m in region II is related to the velocity V in region I by $V_m = (k_m/k)V$. In our new model there will be an increase in volume flux, over that in the old model, of amount $(V_m - V)A_m$, where A_m is the cross-sectional area of region II. Since $Q_0 = \pi R_c^2 V$, and

$$A_m = \pi(R_c - d_f)^2 - \pi(R_c - d_f - d_m)^2 = \pi d_m(2R_c - 2d_f - d_m) \doteq 2\pi R_c d_m,$$

we now have, in place of Eq. 3,

$$\frac{Q}{Q_0} = 1 - \frac{d_f}{R_c} F(\alpha, d_f/k_m) + 2(k_m/k - 1)\frac{d_m}{R_c}. \quad (14)$$

The last term in this equation is positive and represents a relaxing effect of an increase in permeability due to increase in porosity near the wall. The second to last term represents primarily a restricting effect due to no-slip at the wall.

CONCLUSIONS

We have developed a model which involves core, intermediate and wall regions, and in this respect it is similar to that of Cohen and Metzner, but is otherwise distinctly different. We have derived an expression for Q/Q_0 (Eq. 14) which is essentially linear in the small parameters d_f/R_c and d_m/R_c. In this expression the restricting and relaxing contributions to the wall effect are clearly displayed. The parameters d_f, d_m and k_m/k may be varied (within limits to fit experimental data. We stop short of doing so here, because our purpose is not to compete directly with the theory of Cohen and Metzner, which already ties in quite well with experiment, but rather to provide an additional vantage point from which to view that theory.

NOTATION

- A = constant of integration, given by Eq. 10
- A_m = cross-sectional area of intermediate layer
- d_f, d_m = thickness of fluid layer, intermediate layer, respectively
- F = function given by Eq. 12
- k = permeability of bulk of porous medium
- k_m = permeability of intermediate layer
- L = length of column
- p = pressure
- Δp = pressure drop across length L of column
- Q, Q_0 = volume flux through column in presence, absence (respectively) of wall effect
- R = radial distance
- R_c = radius of cross-section of column
- R_m = radius of interface ($R_m = R_c - d_f$)
- r = dimensionless radial distance
- U = velocity of fluid
- $\tilde{U}$ = velocity scale ($\tilde{U} = R_c^2 \Delta p/4\mu L$)
- u = dimensionless velocity of fluid
- V = superficial velocity
- V_m = superficial velocity in intermediate region
- v = dimensionless superficial velocity

Greek Letters

- α = Beavers-Joseph nondimensional constant
- β = $4\alpha k^{1/2}/R_c$
- γ = $d_f/k^{1/2}$
- ϵ = $\gamma k^{1/2}/R_c$
- η = $1 - \epsilon$
- λ = $\alpha R\ /k^{1/2}$
- μ = dynamic viscosity of fluid

c

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Non-Newtonian Fluid-Particle Mass Transfer in Granular Beds

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There is a wealth of experimental data and empirical correlations for mass transfer in Newtonian fluids flowing through granular beds, but only a few studies on the effect of non-Newtonian flow characteristics have been published so far.

Bhavaraju et al. (1978) and Kawase and Ulbrecht (1981) treated mass transfer in non-Newtonian flows through multiparticle systems by using a cell model. The applicability of their results is, however, restricted to very low Reynolds numbers. No models for high Reynolds number region have been reported so far.

The objective of this note is to discuss the effects of non-Newtonian flow behavior at high Reynolds numbers on the mass transfer in granular beds using the boundary layer model which was applied successfully to the case of Newtonian fluids by Carberry (1960). The development of the model starts with considering